

A novel route to a finite center-of-mass momentum pairing state; current driven FFLO state

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(Dated: February 6, 2008)

The previously studied Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state is stabilized by a magnetic field via the Zeeman coupling in spin-singlet superconductors. Here we suggest a novel route to achieve non-zero center-of-mass momentum pairing states in superconductors with Fermi surface nesting. We investigate two-dimensional superconductors under a uniform external current, which leads to a finite pair-momentum of \mathbf{q}_e . We find that an FFLO state with a spontaneous pair-momentum of \mathbf{q}_s is stabilized above a certain critical current which depends on the direction of the external current. A finite \mathbf{q}_s arises in order to make the total pair-momentum of $\mathbf{q}_t (= \mathbf{q}_s + \mathbf{q}_e)$ perpendicular to the nesting vector, which lowers the free energy of the FFLO state, as compared to the superconducting and normal states. We also suggest experimental signatures of the FFLO state.

PACS numbers: 74.25.Dw, 74.25.Sv, 74.81.-g

Introduction: Fulde and Ferrell[1], and Larkin and Ovchinnikov[2] predicted that Cooper pairs with a nonzero center-of-mass momentum can be stabilized under a finite magnetic field via the Zeeman coupling, when the Zeeman term dominates over the orbital effect. Recently, there has been growing interest in theoretical and experimental studies of the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state in spin-singlet superconductors and trapped cold fermionic atoms with unequal densities[3]. It has been suggested theoretically that the organic superconductors like κ -(ET)₂ salts and λ -(ET)₂ salts, heavy-fermion superconductors, and high T_C cuprates are promising candidates for observing the FFLO state. The formation of a possible FFLO state has been very recently inferred from specific heat[4] and magnetization measurements [5] on one of heavy fermion systems, CeCoIn₅ known to be a d -wave superconductor.

The main idea of the previously studied FFLO state is based on the well known BCS theory that Cooper pairs in spin-singlet superconductors are made of fermions with opposite spins. When the magnetic field is acting on the spin of the electrons via the Zeeman coupling, electrons tend to polarize along the direction of the magnetic field to gain polarization energy, while the pairing of opposite spins is favorable for condensation energy. As a result of the competition, the superconducting state undergoes a transition to the FFLO state, and the system eventually enters the normal state, as the magnetic field is further increased. It has been shown that the FFLO state can lead to an enhancement of the critical magnetic field of up to 2.5 times the Pauli paramagnetic limit. It has also been suggested that the FFLO state can be realized when two species of fermion have different densities in trapped cold fermionic atoms.[3]

In this paper, we suggest a novel route to achieve the FFLO state. We study two dimensional superconductors in the presence of uniform external current, \mathbf{j}_e . When a

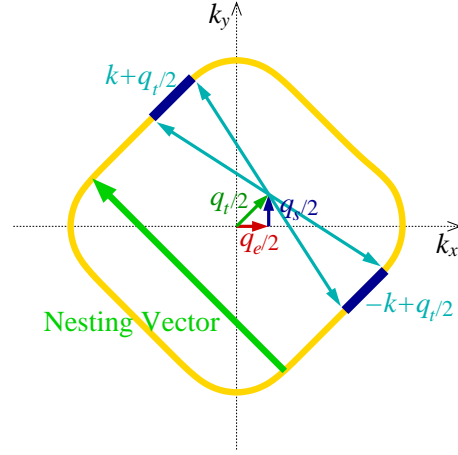


FIG. 1: (Color online) The thick arrow indicates a nesting vector of a Fermi surface on a square lattice. When an external current with a momentum \mathbf{q}_e is applied along the x -axis, a spontaneous pair-momentum of \mathbf{q}_s is generated in such a way that the total pair-momentum, $\mathbf{q}_t (= \mathbf{q}_e + \mathbf{q}_s)$ is perpendicular to the nesting vector. The region of Cooper pairing between electrons with momentum $\mathbf{k} + \mathbf{q}_t/2$ and $-\mathbf{k} + \mathbf{q}_t/2$ is indicated by the dark bars. This pairing on the Fermi surface has the dominant contribution to lowering the free energy of the FFLO state, as compared to the superconducting and normal states.

current flows, Cooper pairs acquire a finite external pair-momentum of \mathbf{q}_e , and the superconducting state becomes unstable towards the normal state at a critical current. If the underlying Fermi surface has nesting, we find that the strength of this critical current depends on the direction of the current. In the case of s -wave superconductors, the critical current is maximal when the current is perpendicular to the nesting vector. We show that the FFLO state with a spontaneous pair-momentum of \mathbf{q}_s is stabilized, when the current is not perpendicular to the nesting vectors. For example, a spontaneous pair-momentum, \mathbf{q}_s is

induced perpendicular to the current, when the current is along the x -axis of the square lattice as shown in Fig. 1. The total pair-momentum of $\mathbf{q}_t (= \mathbf{q}_e + \mathbf{q}_s)$ is determined to maximize the Cooper pairing between electrons with momentum $\mathbf{k} + \mathbf{q}_t/2$ and $-\mathbf{k} + \mathbf{q}_t/2$ (in a lab frame). This pairing on the Fermi surface has the dominant contribution to lowering the free energy of the FFLO state, as compared to the superconducting and normal states.

Superconductors in the presence of a uniform supercurrent: We consider a sample with thickness of $d \ll \xi$, where ξ is the coherence length. Under this condition, the magnitude of the superconducting gap and the current are uniform across the sample. When a uniform current flows, the Cooper-pair acquires a center-of-mass momentum of \mathbf{q}_e . For an infinitesimal current there will be no contribution from the quasiparticles, and the supercurrent will be simply proportional to q_e . However, the change of the quasiparticle spectrum in the presence of a supercurrent known as a Doppler shift, is written as follows.

$$E_{\mathbf{k}} = E_{\mathbf{k}}^0 + \mathbf{v}_{\mathbf{k}} \cdot \mathbf{q}_e/2 \quad (1)$$

where $E_{\mathbf{k}}^0$ is the quasiparticle spectrum in the absence of the current, and $\mathbf{v}_{\mathbf{k}}$ is the quasiparticle velocity. The total supercurrent as a function of \mathbf{q}_e rises linearly at small \mathbf{q}_e and reaches a maximum at a certain value of \mathbf{q}_e . This value determines the critical current at which the superconducting state become unstable towards the normal state. Accordingly, a uniform supercurrent reduces the amplitude of the superconducting order parameter, which has been extensively discussed in nodal superconductors.[6, 7]

One can intuitively understand that Cooper pairs with a spontaneous center-of-mass momentum would not occur, if the quasiparticle velocity is constant around the Fermi surface, for s -wave superconductors due to the Galilean invariance. However, once the quasiparticle velocity strongly depends on the angle around the Fermi surface due to Fermi surface nesting, the strength of the critical current would vary as a function of the angle, ϕ . This anisotropy of the critical current is an essential ingredient for an existence of the FFLO state. In the case of anisotropic order parameters, such as an extended d -wave order parameter, the anisotropic critical current occurs even for a circular Fermi surface. However, we found that the region of the FFLO state for this case is too narrow to be practically realizable.

In Fig. 2, we show the direction dependence of the critical current for a s -wave superconductor with the tight-binding electronic dispersion of the square lattice. The strength of the critical current strongly depends on its direction. It attains its maximum, j_C^{\max} , when the current is perpendicular to the nesting vector. When the current is applied along x -direction where the critical current has its minimum, j_C^{\min} , it is possible to develop a

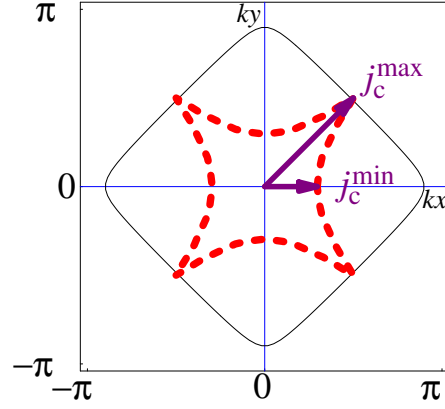


FIG. 2: (Color online) The direction dependence of the critical current value (in arbitrary unit) is shown as the thick dashed line. The thin solid line is the Fermi surface of the tight binding model on the square lattice. The critical current has its maximum, j_C^{\max} when the current is perpendicular to the nesting vector, while its minimum is j_C^{\min} along the x (or y)-direction.

spontaneous pair-momentum of \mathbf{q}_s to extend the superconducting phase, even above j_C^{\min} . The development of the FFLO state leads to an enhancement of the critical current, similar to the enhancement of the critical magnetic field in the previously studied FFLO phase. Below we carry out a free energy computation to find a possible FFLO state and present temperature and current phase diagrams.

Free energy and Phase diagram: Here we adopt the standard BCS formalism to compute the free energy, and determine the phase diagram as a function of temperature and external current. The single particle Green function $G(i\omega_n, \mathbf{k})$ in the presence of finite superflow is given by[8]

$$G^{-1}(i\omega_n, \mathbf{k}) = i\omega_n - \mathbf{v}_{\mathbf{k}} \cdot \mathbf{q}_t/2 + \xi_{\mathbf{k}}\rho_3 + \Delta(\mathbf{k})\rho_1\sigma_1, \quad (2)$$

where the single particle dispersion on the square lattice, $\xi_{\mathbf{k}} = -t[\cos(k_x) + \cos(k_y)] - \mu$. The superconducting order parameter, $\Delta(\mathbf{k}) = \Delta_0 f(\mathbf{k})$, where $f(\mathbf{k})$ represents the relative momentum dependence of the order parameter. ρ_i and σ_i are Pauli matrices in particle-hole space and spin space, respectively.

The free energy is then given by

$$\begin{aligned} F - \mu N = & - \sum_{\mathbf{k}} \frac{\Delta_{\mathbf{k}}^2}{E_{\mathbf{k}}^0} [f(E_{\mathbf{k}+}) + f(E_{\mathbf{k}-}) - 1] \\ & - \sum_{\mathbf{k}\mathbf{k}'} \frac{V_{\mathbf{k}\mathbf{k}'}}{4} \frac{\Delta_{\mathbf{k}}\Delta_{\mathbf{k}'}}{E_{\mathbf{k}}^0 E_{\mathbf{k}'}^0} (1 - f(E_{\mathbf{k}+}) - f(E_{\mathbf{k}-})) \\ & \times (1 - f(E_{\mathbf{k}'+}) - f(E_{\mathbf{k}'-})) \\ & + \sum_{\mathbf{k}} \left[(\xi_{\mathbf{k}} - E_{\mathbf{k}}^0) \right. \\ & \left. + \frac{\ln(1 + f(E_{\mathbf{k}+}))(1 + f(E_{\mathbf{k}-}))}{\beta} \right], \quad (3) \end{aligned}$$

where $E_{\mathbf{k}\pm} = E_{\mathbf{k}}^0 \pm \mathbf{v}_{\mathbf{k}} \cdot \mathbf{q}_t/2 = \sqrt{\xi_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2} \pm \mathbf{v}_{\mathbf{k}} \cdot \mathbf{q}_t/2$. The total pair-momentum, \mathbf{q}_t is given by the sum of the external and spontaneous pair-momenta, $\mathbf{q}_t = \mathbf{q}_e + \mathbf{q}_s$. The pairing interaction $V_{\mathbf{k}\mathbf{k}'}$ is assumed to have the form of $V_0 f(\mathbf{k})f(\mathbf{k}')$.

When an external current is applied along the parallel axes of the square lattice, $\mathbf{q}_e = q_e \hat{x}$ (or \hat{y}), we find that the minimum of the free energy as a function of the external current shifts from $\mathbf{q}_s = 0$ to a finite value of \mathbf{q}_s above a critical value of q_e^{C1} . The direction of \mathbf{q}_s is perpendicular to the applied current in order to keep constant input and output currents along the direction of the applied current. The magnitude of \mathbf{q}_s depends on the magnitude of the external current \mathbf{q}_e , and it is spontaneously arises so as to maximize the regions of electron pairing as indicated by the dark bars in Fig. 1.

The result of the free energy near zero temperature as a function of the external current, q_e is shown in Fig. 3 for several values of q_s/q_e for the case of s -wave superconductors. Note that above a critical value of q_e^{C1} , the solution with a finite q_s has lower energy than that of the uniform superconducting state. It is also important to note that the superconducting state becomes unstable towards the normal state when q_e exceeds the critical value of q_e^{C2} , where the normal state has the lowest energy. While our result of Fig. 3 is shown for a s -wave superconductor $f(\mathbf{k}) = 1$, the qualitative behavior of the phase transition to the FFLO state in the presence of current does not depend on the detailed nature of the pairing symmetry, $f(\mathbf{k})$.

The temperature-current phase diagrams for s - and $d_{x^2-y^2}$ -wave superconductors are shown in Fig. 4. The solid line is the second order phase boundary separating the normal state and the superconducting state. For $T/T_C < 0.7$ (and 0.6 for a d -wave case), the FFLO state which is bounded by the dashed and solid lines is stabilized. The dotted line indicates the phase boundary between the normal and superconducting states in the absence of the FFLO state, where the critical current at $T = 0$ is denoted by q_{C0} . Note that the FFLO state enhances the critical current about 1.7 times for $\mu/(2t) = 0.05$ at $T = 0$ for a s -wave superconductor. The transition between the FFLO and superconducting states is first order for both s - and d -wave superconductors at low temperatures. The inset shows the magnitude of the spontaneous pair-momentum, q_s normalized by the external pair-momentum q_e as a function of q_e/q_{C0} . We have also analyzed the free energy for a p -wave superconductor with $f(\mathbf{k}) = \sin k_x \pm i \sin k_y$, and found that the qualitative features of temperature-current phase diagram resemble those for the s - and d -wave cases shown in Fig. 4.

Experimental signatures: Here we assume that the superconducting gap and supercurrent will be uniform across the system. This condition can be satisfied for a sample with the thickness of $d \ll \xi$, where ξ is the

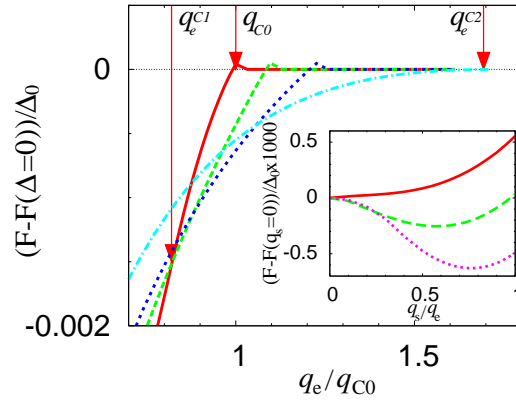


FIG. 3: (Color online) The free energy near zero temperature vs. q_e for various q_s/q_e . q_e is normalized by q_{C0} which is the critical current in the absence of the FFLO state. The solid, dashed, dotted, and dot-dashed lines are for $q_s/q_e = 0, 0.4, 0.8$ and 1.0 , respectively. Below $q_e^{C1} (= 0.82q_{C0})$, the uniform superconducting state with $q_s = 0$ has the lowest energy, while above $q_e^{C2} (= 1.6935q_{C0})$ the normal state has the lowest energy. The FFLO states with finite values of q_s are stabilized in the window of $q_e^{C1} < q_e < q_e^{C2}$. The inset shows the free energy vs. q_s/q_e for various q_e near the transition between the uniform superconducting and the FFLO states. The solid, dashed, dotted lines are for $q_e/q_{C0} = 0.8197, 0.8852$, and 0.9836 , respectively.

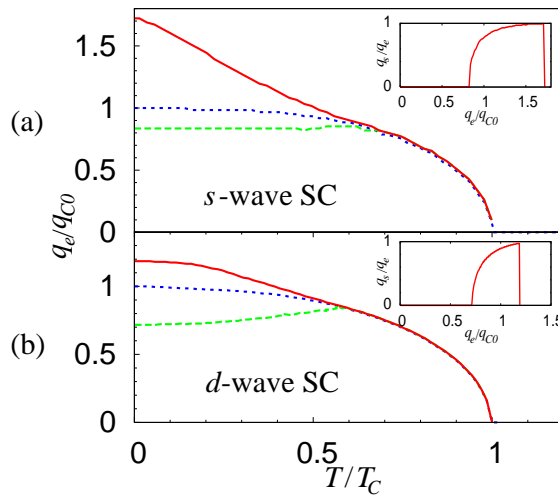


FIG. 4: (Color online) A temperature-current phase diagram for (a) s -wave and (b) $d_{x^2-y^2}$ superconductors. The solid line is the second order phase boundary between the normal and FFLO states, while the dashed line is the phase boundary between the uniform superconducting and FFLO states. The dotted line indicates the phase boundary between the normal and superconducting states in the absence of the FFLO state, where the critical current is denoted by q_{C0} at $T = 0$. Temperature and external current are normalized by the critical temperature of T_C and q_{C0} , respectively. The inset shows the magnitude of the spontaneous pair-momentum q_s normalized by q_e vs. q_e/q_{C0} .

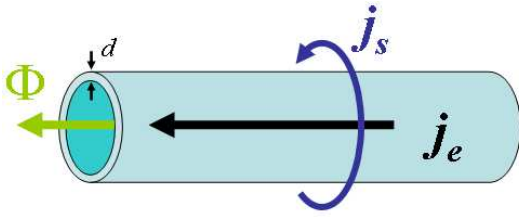


FIG. 5: (Color online) A schematic diagram for an experimental set-up. When an external current is applied along the axial direction of the tube, where the thickness $d \ll \xi$, a spontaneous current will be generated along the azimuthal direction in the FFLO state. Φ denotes the flux induced by the spontaneous current.

coherence length. Below we offer a possible experimental set-up to detect the current driven FFLO state.

Consider a superconductor of cylindrical geometry as shown in Fig. 5. If we apply the current along the axial direction, the spontaneous current, \mathbf{j}_s will be generated along the azimuthal direction in the FFLO state as shown in Fig. 5; one of the two-fold degenerate directions of the spontaneous current will be chosen. The induced current will generate a spontaneous flux, Φ . We find that a unit of flux can be generated when the circumference of the tube is approximately 50 nm ($180 \times$ lattice spacing) for s -wave superconductors near half-filling with the tight-binding electronic dispersion when $q_s/q_e \approx 1$.

The boundary conditions of an order parameter for finite size samples deserve some discussion. We suggested the cylinder shape of the sample in the above experiment, so that an order parameter with a finite current, $\Delta(\mathbf{R}) \propto e^{i\mathbf{q}_s \cdot \mathbf{R}}$ can be stabilized, assuming that we can neglect finite size effects on the length of the tube. Depending on boundary, a linear combination of two plane waves, such as $\Delta(\mathbf{R}) \propto \cos(\mathbf{q}_s \cdot \mathbf{R})$, can be a possible solution, which leads to a spatial modulation of the order parameter magnitude with a periodicity of $2\pi/q_s$.

Recently, the FFLO phase with unequal numbers of two hyperfine states of fermionic atoms has been proposed in the context of cold fermionic atoms. The experiments have been carried out using Li atoms across a Feshbach resonance. The experiment showed that paired and unpaired fermions phase separate when the imbalance is large[9], but further experiments are needed to clarify the nature of the state for a small imbalance of densities[10]. We suggest that the current driven FFLO state can be also realized in optical lattices of cold atoms, where tight binding dispersions have been found.

Summary: We studied the possible existence of the FFLO state in the presence of an external current within

BCS theory. We found that the FFLO state with a spontaneous Cooper pair-momentum can be stabilized in the presence of finite currents in superconductors with Fermi surface nesting. The anisotropy of the critical current due to the nesting plays a crucial role for the existence of the FFLO state. A spontaneous pair-momentum is generated to extend the superconducting phase and leads to an enhancement of the critical current for certain directions. The FFLO state can be found in anisotropic superconductors with a circular Fermi surface, but the region of the FFLO state is too narrow to be practically realizable. We suggested a possible experimental set-up to detect a spontaneous flux induced by a spontaneous current, which will be a direct signature of the FFLO state. We also discussed a possible realization of the FFLO state induced by a finite current in optical lattices of cold fermionic atoms.

This work was supported by NSERC of Canada(HD, MS, HYK), Canada Research Chair, Canadian Institute for Advanced Research, and Alfred P. Sloan Research Fellowship(HYK).

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